

## Financial Options and Applications in Corporate Finance

Microsoft has granted options to buy more than 800 million shares to its employees, or about 16,000 options per person. ${ }^{1}$ Of course, many employees have fewer options and some have more, but any way you cut it, that's a lot of options. Microsoft isn't the only company with mega-grants: Bank of America, Citigroup, IBM, JPMorgan Chase, and Ford are among the many companies that have granted options to buy more than 100 million shares to their employees. Whether your next job is with a high-tech firm, a financial service company, or a manufacturer, you will probably receive stock options, so it's important that you understand them.

In a typical grant, you receive options allowing you to purchase shares of stock at a fixed price, which is called the "strike" price, on or before a stated expiration date. Most plans have a vesting period, during which you can't exercise the options. For example,
suppose you are granted 1,000 options with a strike price of $\$ 50$, an expiration date 10 years from now, and a vesting period of 3 years. Even if the stock price rises above $\$ 50$ during the first 3 years, you can't exercise the options due to the vesting requirement. After 3 years, if you are still with the company, you have the right to exercise the options. For example, if the stock goes up to $\$ 110$, you could pay the company $\$ 50(1,000)=\$ 50,000$ and receive 1,000 shares of stock worth $\$ 110,000$. However, if you don't exercise the options within 10 years, they will expire and thus be worthless.

Even though the vesting requirement prevents you from exercising the options the moment they are granted to you, the options clearly have some immediate value. Therefore, if you are choosing between different job offers where options are involved, you will need a way to determine the value of the alternative options. This chapter explains how to value options, so read on.

[^0]The textbook's Web site contains an Excel file that will guide you through the chapter's calculations. The file for this chapter is FM12 Ch 09 Tool Kit.xls, and we encourage you to open the file and follow along as you read the chapter.

Every manager should understand the basic principles of option pricing. First, many projects allow managers to make strategic or tactical changes in plans as market conditions change. The existence of these "embedded options" often means the difference between a successful project and a failure. Understanding basic financial options can help you manage the value inherent in these real options. Second, many companies use derivatives, which are in essence financial options, to manage risk, so an understanding of financial options is necessary before tackling derivatives. Third, option pricing theory provides insights into the optimal debt/equity choice, especially when convertible securities are involved. And fourth, understanding financial options will help you deal better with any employee stock options that you receive.

### 9.1 Financial Options

An option is a contract that gives its holder the right to buy (or sell) an asset at some predetermined price within a specified period of time. The following sections explain the different features that affect an option's value.

## Option Types and Markets

There are many types of options and option markets. ${ }^{2}$ To illustrate how options work, suppose you owned 100 shares of General Computer Corporation (GCC), which on January 9, 2007, sold for $\$ 53.50$ per share. You could sell to someone the right to buy your 100 shares at any time until May 18, 2007, at a price of, say, $\$ 55$ per share. This is called an American option, because it can be exercised any time before it expires. By contrast, a European option can only be exercised on its expiration date. The $\$ 55$ is called the strike, or exercise, price. The last day that the option can be exercised is called the expiration date. Such options exist, and they are traded on a number of exchanges, with the Chicago Board Options Exchange (CBOE) being the oldest and the largest. This type of option is defined as a call option, because the buyer has a "call" on 100 shares of stock. The seller of an option is called the option writer. An investor who "writes" call options against stock held in his or her portfolio is said to be selling covered options. Options sold without the stock to back them up are called naked options. When the strike price exceeds the current stock price, a call option is said to be out-of-the-money. When the strike price is less than the current price of the stock, the option is in-the-money.

You can also buy an option that gives you the right to sell a stock at a specified price within some future period-this is called a put option. For example, suppose you think GCC's stock price is likely to decline from its current level of $\$ 53.50$ sometime during the next 4 months. A put option will give you the right to sell at a fixed price even after the market price declines. You could then buy at the new lower market price, sell at the higher fixed price, and earn a profit. Table 9-1 provides data on GCC's options. You could buy the 4-month May put option for $\$ 218.75(\$ 23 / 16 \times 100)$. That would give you the right to sell 100 shares (that you would not necessarily own) at a price of $\$ 50$ per share ( $\$ 50$ is the strike price). Suppose you bought this 100-share contract for $\$ 218.75$ and then GCC's stock fell

[^1]| Closing Price | Strike Price | CALLS-LAST QUOTE |  |  | PUTS-LAST QUOTE |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | February | March | May | February | March | May |
| General Computer Corporation (GCC) |  |  |  |  |  |  |  |
| 531/2 | 50 | 41/4 | 43/4 | 51/2 | 5/8 | $13 / 8$ | 23/16 |
| 531/2 | 55 | 15/16 | 21/16 | 31/8 | 25\% | $r$ | 41/2 |
| 531/2 | 60 | 5/16 | 11/16 | $11 / 2$ | 65\% | $r$ | 8 |
| U.S. Medical |  |  |  |  |  |  |  |
| 565/8 | 55 | $41 / 4$ | 51/8 | 7 | 21/4 | $33 / 4$ | $r$ |
| Sport World |  |  |  |  |  |  |  |
| 531/8 | 55 | 1/2 | 11/8 | $r$ | 21/8 | $r$ | $r$ |
| Note: r means not traded on January 9. |  |  |  |  |  |  |  |

to $\$ 45$. You could buy the stock on the open market at $\$ 45$ and exercise your put option by selling the stock at $\$ 50$. Your profit from exercising the option would be $(\$ 50-\$ 45)(100)=\$ 500$. After subtracting the $\$ 218.75$ you paid for the option, your profit (before taxes and commissions) would be $\$ 281.25$.

Table 9-1 contains an extract from the Listed Options Quotations Table as it would appear the next day in a daily newspaper. Sport World's February $\$ 55$ call option sold for $\$ 0.50$. Thus, for $\$ 0.50(100)=\$ 50$ you could buy options that would give you the right to buy 100 shares of Sport World stock at a price of $\$ 55$ per share from January until February, or during the next month. ${ }^{3}$ If the stock price stayed below $\$ 55$ during that period, you would lose your $\$ 50$, but if it rose to $\$ 65$, your $\$ 50$ investment would increase in value to $(\$ 65-\$ 55)(100)=\$ 1,000$ in less than 30 days. That translates into a very healthy annualized rate of return. Incidentally, if the stock price did go up, you would not actually exercise your options and buy the stock-rather, you would sell the options, which would then have a value of at least $\$ 1,000$ versus the $\$ 50$ you paid, to another option buyer or to the original seller.

In addition to options on individual stocks, options are also available on several stock indexes such as the NYSE Index and the S\&P 100 Index. Index options permit one to hedge (or bet) on a rise or fall in the general market as well as on individual stocks.

Option trading is one of the hottest financial activities in the United States. The leverage involved makes it possible for speculators with just a few dollars to make a fortune almost overnight. Also, investors with sizable portfolios can sell options against their stocks and earn the value of the option (less brokerage commissions), vides 20-minute delayed quotes for equity, index, and LEAPS options at http://www.cboe.com.

[^2]even if the stock's price remains constant. Most important, though, options can be used to create hedges that protect the value of an individual stock or portfolio. ${ }^{4}$

Conventional options are generally written for 6 months or less, but a type of option called a Long-Term Equity AnticiPation Security (LEAPS) is different. Like conventional options, LEAPS are listed on exchanges and are available on both individual stocks and stock indexes. The major difference is that LEAPS are long-term options, having maturities of up to $21 / 2$ years. One-year LEAPS cost about twice as much as the matching 3-month option, but because of their much longer time to expiration, LEAPS provide buyers with more potential for gains and offer better long-term protection for a portfolio.

Corporations on whose stocks options are written have nothing to do with the option market. Corporations do not raise money in the option market, nor do they have any direct transactions in it. Moreover, option holders do not vote for corporate directors or receive dividends. There have been studies by the SEC and others as to whether option trading stabilizes or destabilizes the stock market, and whether this activity helps or hinders corporations seeking to raise new capital. The studies have not been conclusive, but option trading is here to stay, and many regard it as the most exciting game in town.

Table 9-1 can provide some insights into call option valuation. First, we see that at least three factors affect a call option's value:

1. Market price versus strike price. The higher the stock's market price in relation to the strike price, the higher will be the call option price. Thus, Sport World's $\$ 55$ February call option sells for $\$ 0.50$, whereas U.S. Medical's $\$ 55$ February option sells for $\$ 4.25$. This difference arises because U.S. Medical's current stock price is $\$ 565 \%$ versus only $\$ 531 /$ for Sport World.
2. Level of strike price. The higher the strike price, the lower the call option price. Thus, all of GCC's call options, regardless of exercise month, decline as the strike price increases.
3. Length of option. The longer the option period, the higher the option price. This occurs because the longer the time before expiration, the greater the chance that the stock price will climb substantially above the exercise price. Thus, option prices increase as the expiration date is lengthened.

Other factors that affect option values, especially the volatility of the underlying stock, are discussed in later sections.

## Exercise Value versus Option Price

How is the actual price of a call option determined in the market? In a later section, we present a widely used model (the Black-Scholes model) for pricing call options, but first it is useful to establish some basic concepts. To begin, we define a call option's exercise value as follows: ${ }^{5}$

$$
\text { Exercise value = MAX[Current price of the stock - Strike price, } 0] .
$$

[^3]Space Technology Inc.: Option Price and Exercise Value


The exercise value is what the option would be worth if it expired immediately. For example, if a stock sells for $\$ 50$ and its option has a strike price of $\$ 20$, then you could buy the stock for $\$ 20$ by exercising the option. You would own a stock worth $\$ 50$, but you would have to pay only $\$ 20$. Therefore, the option would be worth $\$ 30$ if you had to exercise it immediately. The minimum exercise value is zero, because no one would exercise an out-of-the-money option.

Figure 9-1 presents some data on Space Technology Inc. (STI), a company that recently went public and whose stock price has fluctuated widely during its short history. The third column in the tabular data shows the exercise values for STI's call option when the stock was selling at different prices; the fourth column gives the actual market prices for the option; and the fifth column
shows the time value, which is the excess of the actual option price over its exercise value. ${ }^{6}$

First, notice that the market value of the option is zero when the stock price is zero. This is because a stock price falls to zero only when there is no possibility that the company will ever generate any future cash flows; in other words, the company must be out of business. In such a situation, an option would be worthless.

Second, notice that the market price of the option is always greater than or equal to the exercise value. If the option price ever fell below the exercise value, then you could buy the option and immediately exercise it, reaping a riskless profit. Because everyone would try to do this, the price of the option would be driven up until it was at least as high as the exercise value.

Third, notice that the market value of the option is greater than zero even when the option is out-of-the-money. For example, the option price is $\$ 2$ when the stock price is only $\$ 10$. Depending on the remaining time until expiration and the stock's volatility, there is a chance that the stock price will rise above $\$ 20$, so the option has value even if it is out-of-the-money.

Fourth, Figure 9-1 shows the value of the option steadily increasing as the stock price increases. This shouldn't be surprising, since the option's expected payoff increases along with the stock price. But notice that as the stock price rises, the option price and exercise value begin to converge, causing the time value to get smaller and smaller. This happens because there is virtually no chance that the stock will be out-of-the-money at expiration if the stock price is presently very high. Thus, owning the option is like owning the stock, less the exercise price. Although we don't show it in Figure 9-1, the market price of the option also converges to the exercise value if the option is about to expire. With expiration close, there isn't much time for the stock price to change, so the option's time value would be close to zero for all stock prices.

Fifth, an option has more leverage than the stock. For example, if you buy STI's stock at $\$ 20$ and it goes up to $\$ 30$, you would have a $50 \%$ rate of return. But if you bought the option instead, its price would go from $\$ 8$ to $\$ 16$ versus the stock price increase from $\$ 20$ to $\$ 30$. Thus, there is a $100 \%$ return on the option versus a $50 \%$ return on the stock. Of course, leverage is a double-edged sword: If the stock price falls to $\$ 10$, then you would have a $50 \%$ loss on the stock, but the option price would fall to $\$ 2$, leaving you with a $75 \%$ loss. In other words, the option magnifies the returns on the stock, for good or ill.

Sixth, options typically have considerable upside potential but limited downside risk. To see this, suppose you buy the option for $\$ 8$ when the stock price is $\$ 20$. If the stock price is $\$ 28$ when the option expires, your net gain would be $\$ 0$ : you gain $\$ 28$ $-\$ 20=\$ 8$ when you exercise the option, but your original investment was $\$ 8$. Now suppose the stock price is either $\$ 30$ or $\$ 20$ at expiration. If it's $\$ 30$, your net gain is $\$ 10-\$ 8=\$ 2$. If it's $\$ 20$, the stock is out-of-the-money, and your net loss is the $\$ 8$ cost of your investment. Now suppose the stock price is either $\$ 50$ or $\$ 5$. If it's $\$ 50$, your net gain is $\$ 30-\$ 8=\$ 22$; if $\$ 5$, your net loss is still your $\$ 8$ initial investment. As this example shows, the payoffs from the option aren't symmetric. The most you can lose is $\$ 8$, and this happens whether the stock price at expiration is $\$ 20, \$ 10$, or even $\$ 1$. On the other hand, every dollar of stock price above $\$ 20$ yields an extra dollar of payoff from the option, and every dollar above $\$ 28$ is a dollar of net profit.

[^4]When granted to executives and other employees, options are a "hybrid" form of compensation. At some companies, especially small ones, option grants may be a substitute for cash wages-employees are willing to take lower cash salaries if they have options. Options also provide an incentive for employees to work harder. Whether issued to motivate employees or to conserve cash, options clearly have value at the time they are granted, and they transfer wealth from existing shareholders to employees to the extent that they do not reduce cash expenditures or increase employee productivity sufficiently to offset their value at the time of issue.

Companies like the fact that an option grant requires no immediate cash expenditure, although it might dilute shareholder wealth if it is later exercised. Employees, and especially CEOs, like the potential wealth that they receive when they are granted options. When option grants were relatively small, they didn't show up on investors' radar screens. However, as the high-tech sector began making mega-grants in the 1990s, and as other
industries followed suit in the heavy use of options, stockholders began to realize that large grants were making some CEOs filthy rich at the stockholders' expense.

Before 2005, option grants were not very visible in companies' financial reports. Even though such grants are clearly a wealth transfer to employees, companies were only required to footnote the grants and could ignore them when reporting their income statements and balance sheets. The Financial Accounting Standards Board now requires companies to show option grants as an expense on the income statement. To do this, the value of the grant is estimated at the time of the grant and then expensed during the vesting period. For example, if the initial value is $\$ 100$ million and the vesting period is 2 years, the company would report a $\$ 50$ million expense for each of the next 2 years. This approach isn't perfect because it isn't a cash expense, and it does not take into account changes in the option's value after it is initially granted. However, it does make the option grant more visible to investors, which is a good thing.

In addition to the stock price and the exercise price, the price of an option depends on three other factors: (1) the option's term to maturity, (2) the variability of the stock price, and (3) the risk-free rate. We will explain precisely how these factors affect call option prices later, but for now, note these points:

1. The longer a call option has to run, the greater its value and the larger its time value. If an option expires at 4 P.M. today, there is not much chance that the stock price will go up very much, so the option will sell at close to its exercise value and its time value will be small. On the other hand, if the expiration date is a year away, the stock price could rise sharply, pulling the option's value up with it.
2. An option on an extremely volatile stock is worth more than one on a very stable stock. If the stock price rarely moves, then there is little chance of a large gain on the stock; hence the option will not be worth much. However, if the stock is highly volatile, the option could easily become very valuable. At the same time, losses on options are limited-you can make an unlimited amount, but you can lose only what you paid for the option. Therefore, a large decline in a stock's price does not have a corresponding bad effect on option holders. As a result of the unlimited upside but limited downside potential, the more volatile a stock, the higher the value of its options.
3. Options will be exercised in the future, and part of a call option's value depends on the present value of the cost to exercise it. If interest rates are high, then the present value of the cost to exercise is low, which increases the option's value.

Because of Points 1 and 2, a graph such as Figure 9-1 will show that the longer an option's life, the higher its market price line will be above the exercise value line. Similarly, the more volatile the price of the underlying stock, the higher the market price line. We will see precisely how these factors, and also the risk-free rate, affect option values when we discuss the Black-Scholes model.

Define a call option's exercise value. Why is the actual market price of a call option usually above its exercise value?
What are some factors that affect a call option's value?
Brighton Memory's stock is currently trading at $\$ 50$ a share. A call option on the stock with a $\$ 35$ strike price currently sells for $\$ 21$. What are the exercise value and the time value of the call option? (\$15.00; \$6.00)

### 9.2 Introduction to Option Pricing Models: The Binomial Approach

All option pricing models are based on the concept of a riskless hedge. The purpose of such a hedge isn't to create a riskless security-you can buy Treasury securities for that-but, instead, to determine how much an option is worth. To see how this works, suppose a hypothetical investor, we'll call her the hedger, buys some shares of stock and simultaneously writes a call option on the stock. As a result of writing the call option, our hedger (1) receives a payment from the call option's purchaser and (2) assumes an obligation to satisfy the purchaser if he or she chooses to exercise the option. Let's focus only on the hedger's portfolio, which contains stock and the obligation to satisfy the option's purchaser (we'll solve for the amount the hedger receives for selling the option in just a bit). If the stock price goes up, the hedger will earn a profit on the stock. However, the option holder will then exercise the option, our hedger will have to sell a share of stock to the option holder at the strike price (which is below the market price), and that will reduce our hedger's profit on the stock's gain. Conversely, if the stock goes down, our hedger will lose on her stock investment, but she won't lose as much in satisfying the option: If the stock goes down a lot, the option holder won't exercise the option, and the hedger will owe nothing; if the stock goes down a little, then the hedger might still have to sell a share at a below-market price to satisfy the option holder, but the market price will be closer to the strike price, so the hedger will lose less. As we will soon show, it is possible to create the portfolio such that the hedger will end up with a riskless positionthe value of the portfolio will be the same regardless of what the stock does.

If the portfolio is riskless, then its return must be equal to the riskless rate in order to keep the market in equilibrium. If the portfolio offered a higher rate of return than the riskless rate, arbitrageurs would buy the portfolio and in the process

See Web Extension 9A
at the textbook's Web site for more discussion of the binomial approach. the riskless rate. Given the price of the stock, its volatility, the option's exercise price, the life of the option, and the risk-free rate, there is a single option price that satisfies the equilibrium condition, namely, that the portfolio will earn the riskless rate.

The following example applies the binomial approach, so named because we assume the stock price can take on only one of two possible values at the end of each period. The stock of Western Cellular, a manufacturer of cell phones, sells for $\$ 40$ per share. Options exist that permit the holder to buy one share of Western at
an exercise price of $\$ 35$. These options will expire at the end of 1 year. The steps to the binomial approach are shown below.

Step 1. Define the possible ending prices of the stock. Let's assume that Western's stock will be selling at one of two prices at the end of the year, either $\$ 50$ or $\$ 32$. If there is a $70 \%$ chance of the $\$ 50$ price, then Western's expected price is $0.7(\$ 50)+0.3(\$ 32)=\$ 44.6^{7}$ Because the current stock price is $\$ 40$, Western has an $11.5 \%$ expected return: $(\$ 44.6-\$ 40) / \$ 40=$ $0.115=11.5 \%$. If Western were a riskier stock, then we would have assumed different ending prices that had a wider range and possibly a higher expected return. See Web Extension 9A for a more detailed explanation of the relationship between the stock's risk and the possible ending stock prices. Figure 9-2 illustrates the stock's possible price paths and contains additional information that is explained below.
Step 2. Find the range of values at expiration. When the option expires at the end of the year, Western's stock will sell for either $\$ 50$ or $\$ 32$, a range of $\$ 50-\$ 32=\$ 18$. As shown in Figure 9-2, the option will pay $\$ 15$ if the stock is $\$ 50$, because this is above the strike price of $\$ 35$ : $\$ 50-\$ 35=\$ 15$. The option will pay nothing if the stock price is $\$ 32$, because this is below the strike price. The range of option payoffs is $\$ 15-\$ 0=\$ 15$. The hedger's portfolio consists of the stock and the obligation to satisfy the option holder, so the value of the portfolio in 1 year is the stock price minus the option payoff.
Step 3. Buy exactly enough stock to equalize the range of payoffs for the stock and the option. Figure 9-2 shows that the range of payoffs for the stock is $\$ 18$ and the range for the option is $\$ 15$. To construct the riskless portfolio, we need to equalize these ranges so that the profits from the stock exactly offset the losses in satisfying the option holder. We do so by buying $\$ 15 / \$ 18=0.8333$ share and selling one option (or 8,333 shares and 10,000 options).

Here is why equalizing ranges gives the correct number of shares of stock. Let $P_{u}$ be the stock price if it goes up, $\mathrm{P}_{\mathrm{d}}$ the stock price if it goes down, $C_{u}$ the call option payoff if the stock goes up, $C_{d}$ the call option payoff if the stock goes down, and N the number of shares of stock. We want the portfolio value to be the same whether the stock goes up or down. The portfolio value for an up stock price is $N\left(P_{u}\right)-C_{u}$, and the value for a down stock price is $N\left(P_{d}\right)-C_{d}$. Setting these equal and solving for N yields

$$
\begin{equation*}
N=\frac{C_{u}-C_{d}}{P_{u}-P_{d}} \tag{9-1}
\end{equation*}
$$

which is the same as equalizing the ranges. For the stock in this example, we have

$$
\mathrm{N}=\frac{\$ 15-\$ 0}{\$ 50-\$ 32}=0.8333
$$

which is the same answer we got before.

[^5]e-resource
See FM12 Ch 09 Tool
Kit.xls at the textbook's Web site for all calculations.


With $\mathrm{N}=0.8333$, the current value of the stock in the portfolio is $\$ 40(0.8333)=\$ 33.33$. The value of the portfolio's stock at the end of the year will be either $\$ 50(0.8333)=\$ 41.67$ or $\$ 32(0.8333)=\$ 26.67$. As shown in Figure 9-3, the range of the stock's ending value is now $\$ 41.67-\$ 26.67=\$ 15$.
Step 4. Create a riskless hedged investment. We created a riskless portfolio by buying 0.8333 share of the stock and selling one call option, as shown in Figure 9-3. The stock in the portfolio will have a value of either $\$ 41.67$ or $\$ 26.67$, depending on the ending price of Western's stock. The call option that was sold will have no effect on the value of the portfolio if Western's price falls to $\$ 32$ because it will not be exercised-it will expire worthless. However, if the stock price ends at $\$ 50$, the holder of the option will exercise it, paying the $\$ 35$ strike price for stock that would cost $\$ 50$ on the open market. The option holder's profit is the option writer's loss, so the option will cost the hedger $\$ 15$. Now note that the value of the portfolio is $\$ 26.67$ regardless of whether Western's stock goes up or down, so the portfolio is riskless. A hedge has been created that protects against both increases and decreases in the price of the stock.
Step 5. Find the call option's price. To this point, we have not mentioned the price of the call option that was sold to create the riskless hedge. What is the fair, or equilibrium, price? The value of the portfolio will be $\$ 26.67$ at the end of the year, regardless of what happens to the price of the stock. This $\$ 26.67$ is riskless, and so the portfolio should earn the risk-free rate, which is $8 \%$. If the risk-free rate is compounded daily, the present value of the portfolio's ending value is ${ }^{8}$

$$
\mathrm{PV}=\frac{\$ 26.67}{\left(1+\frac{0.08}{365}\right)^{365}}=\$ 24.62
$$

This means that the current value of the portfolio must be $\$ 24.62$ to ensure that the portfolio earns the risk-free rate of return. The current value of the portfolio is equal to the value of the stock minus the value of the obligation to cover the call option. At the time the call option is sold, the obligation's value is exactly

[^6]The Hedge Portfolio


Notes:
${ }^{\circ}$ The portfolio contains 0.8333 share of stock, with a stock price of $\$ 40$, so its value is $0.8333(\$ 40)=\$ 33.33$.
${ }^{\text {b }}$ The ending stock price is $\$ 50$, so the value is $0.8333(\$ 50)=\$ 41.67$.

equal to the price of the option. Because Western's stock is currently selling for $\$ 40$, and because the portfolio contains 0.8333 share, the value of the stock in the portfolio is $0.8333(\$ 40)=\$ 33.33$. What remains is the price of the option:

$$
\begin{aligned}
\text { PV of portfolio } & =\text { Current value of stock in portfolio }- \text { Current option price } \\
\text { Current option price } & =\text { Current value of stock in portfolio }- \text { PV of portfolio } \\
& =\$ 33.33-\$ 24.62=\$ 8.71 .
\end{aligned}
$$

If this option sold at a price higher than $\$ 8.71$, other investors could create riskless portfolios as described above and earn more than the riskless rate. Investors (especially the large investment banking firms) would create such portfolios and sell options until their price fell to $\$ 8.71$, at which point the market would be in equilibrium. Conversely, if the options sold for less than $\$ 8.71$, investors would create an "opposite" portfolio by buying a call option and selling short the stock. ${ }^{9}$ The resulting supply shortage would drive the price up to $\$ 8.71$. Thus, investors (or arbitrageurs) would buy and sell in the market until the options were priced at their equilibrium level.

Clearly, this example is unrealistic. Although you could duplicate the purchase of 0.8333 share by buying 8,333 shares and selling 10,000 options, the stock price assumptions are unrealistic; Western's stock price could be almost anything after 1 year, not just $\$ 50$ or $\$ 32$. However, if we allowed the stock to move up or down more often during the year, then a more realistic range of ending prices would result. For

[^7]See FM 12 Ch 09 Tool
Kit.xls for all calculations. Also, Web Extension 9A provides more discussion of the binomial approach.

The Binomial Lattice

See Web Extension 9A for more discussion of the binomial approach and the Web 9A worksheet in FM12 Ch 09 Tool Kit.xls for all calculations.

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There are many free binomial option-pricing programs on the Web, including one at http://
www.hoadley.net/ options/calculators.htm.

|  | A | B | C | D | E | F |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 147 | Current $\frac{\text { stock price }}{\$ 40.00}$ |  |  |  |  |  |
| 148 |  |  |  |  |  |  |
| 149 |  |  |  |  |  |  |
| 150 |  |  |  |  |  |  |
| 151 |  |  |  |  |  |  |
| 152 |  |  |  |  |  |  |
| 153 |  |  |  |  |  |  |
| 154 |  |  |  |  |  |  |
| 155 |  |  |  |  |  |  |
| 156 |  |  |  |  |  |  |
| 157 |  |  |  |  |  |  |
| 158 |  |  |  |  |  |  |
| 159 |  |  |  |  |  |  |
| 160 |  |  |  |  |  |  |

example, suppose we allowed stock prices to change every 6 months, with Western's stock price either going up to $\$ 46.84$ or down to $\$ 34.16$. If the price goes up in the first 6 months to $\$ 46.84$, then suppose it either goes up to $\$ 54.84$ or down to $\$ 40$ by the end of the year. If the price falls to $\$ 34.16$ during the first 6 months, then suppose it either goes up to $\$ 40$ or down to $\$ 29.17$ by the end of the year. This pattern of stock price movements is called a binomial lattice and is shown in Figure 9-4.

If we focus only on the upper right portion of the lattice shown inside the oval, it is very similar to the problem we just solved in Figures 9-2 and 9-3. We can apply the same solution procedure to find the value of the option at the end of 6 months, given a 6 -month stock price of $\$ 46.84$. As explained in Web Extension 9A, the value of the option at the end of 6 months is $\$ 13.21$, given that the stock price goes up to $\$ 46.84$; see the Web $9 A$ worksheet in FM12 Ch 09 Tool Kit.xls for all calculations. Applying the same approach to the lower right portion of the lattice, the Web Extension and Tool Kit show that the option value at the end of 6 months is $\$ 2.83$, given a 6 -month stock price of $\$ 34.16$. These values are shown in Figure 9-5. Using the values in Figure 9-5 and the same approach as before, we can calculate the current price, which is $\$ 8.60$. Notice that by solving three binomial problems, we are able to find the current option price.

If we break the year into smaller periods and allow the stock price to move up or down more often, the lattice would have a more realistic range of possible outcomes. Of course, estimating the current option price would require solving lots of binomial problems within the lattice, but each problem is very simple, and computers can solve them rapidly. With more outcomes, the resulting estimated option price is more accurate. For example, if we divide the year into 10 periods, the estimated price is $\$ 8.38$. With 100 periods, the price is $\$ 8.41$. With 1,000 , it is still $\$ 8.41$, which shows that the solution converges to its final value with a relatively small number of steps. In fact, as we break the year into smaller and smaller periods, the solution for the binomial approach converges to the Black-Scholes solution, which is described in the next section.

The binomial approach is widely used to value options with more complicated payoffs than the call option in our example, such as employee stock options. This is beyond the scope of a financial management textbook, but if you are interested in learning more about the binomial approach, take a look at the textbooks by Don Chance or John Hull. ${ }^{10}$

[^8]Six-Month Stock Prices and Option Values for the First Section of the Binomial Lattice

|  | A | B | C | D | E | F |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 364 | Current $\frac{\text { stock price }}{\$ 40.00}$ | Current option price \$8.60 |  | $\begin{aligned} & \begin{array}{l} \text { Six-month } \\ \text { stock price } \end{array} \\ & \$ 46.84 \end{aligned}$ | $\begin{gathered} \begin{array}{c} \text { Six-month } \\ \text { option value } \end{array} \\ \$ 13.21 \end{gathered}$ |  |
| 365 |  |  |  |  |  |  |
| 366 |  |  |  |  |  |  |
| 367 |  |  |  |  |  |  |
| 368 |  |  |  |  |  |  |
| 369 |  |  |  |  |  |  |
| 370 |  |  |  |  |  |  |
| 371 |  |  |  |  |  |  |
| 372 |  |  |  |  |  |  |
| 373 |  |  |  |  |  |  |
| 374 |  |  |  | Six-month | Six-month |  |
| 375 |  |  |  | $\frac{\text { stock price }}{\$ 34.16}$ | $\frac{\text { option value }}{\$ 2.83}$ |  |
| 376 <br> 377 |  |  |  |  | \$2.83 |  |

## SELF-TEST

Describe how a risk-free porifolio can be created using stocks and options.
How can such a porifflio be used to help estimate a call option's value?
Left Incorporated's stock price is now $\$ 50$ but it is expected to either go up to $\$ 75$ or down to $\$ 35$ by the end of the year. There is a call option on Lett's stock with a strike price of $\$ 55$ and an expiration date 1 year from now. What are the call option's payoffs if the stock price goes up? If the stock price goes down? If we sell one call option, how many shares of Lett's stock must we buy to create a riskless hedged porffolio consisting of the option position and the stock? What is the payoff of this porffolio? If the annual risk free rate is $6 \%$, how much is the riskless portfolio worth today (assuming daily compounding)? What is the current value of the call option? (\$20; $\$ 0 ; 0.5 ; \$ 17.50 ; \$ 16.48 ; \$ 8.52$ )

### 9.3 The Black-Scholes Option Pricing Model (OPM)

The Black-Scholes Option Pricing Model (OPM), developed in 1973, helped give rise to the rapid growth in options trading. This model, which has even been programmed into some handheld and Web-based calculators, is widely used by option traders.

## OPM Assumptions and Equations

In deriving their option pricing model, Fischer Black and Myron Scholes made the following assumptions:

1. The stock underlying the call option provides no dividends or other distributions during the life of the option.
2. There are no transaction costs for buying or selling either the stock or the option.
3. The short-term, risk-free interest rate is known and is constant during the life of the option.
4. Any purchaser of a security may borrow any fraction of the purchase price at the short-term, risk-free interest rate.
5. Short selling is permitted, and the short seller will receive immediately the full cash proceeds of today's price for a security sold short.

## แயய

For a Web-based option calculator, see hitp://www .cboe.com/LearnCenter/ OptionCalculator.aspx.

## ルபルリル

Robert＇s Online Option Pricer can be accessed at http：／／www．intrepid．com／ ～robertl／optionpricer．html． The site is designed to provide a financial serv－ ice over the Internet to small investors for option pricing，giving anyone a means to price option trades without having to buy expensive software and hardware．

6．The call option can be exercised only on its expiration date．
7．Trading in all securities takes place continuously，and the stock price moves randomly．

The derivation of the Black－Scholes model rests on the concept of a riskless hedge such as the one we set up in the last section．By buying shares of a stock and simultaneously selling call options on that stock，an investor can create a risk－free investment position，where gains on the stock will exactly offset losses on the option．This riskless hedged position must earn a rate of return equal to the risk－ free rate．Otherwise，an arbitrage opportunity will exist，and people trying to take advantage of this opportunity will drive the price of the option to the equilibrium level as specified by the Black－Scholes model．

The Black－Scholes model consists of the following three equations：

$$
\begin{align*}
\mathrm{V} & =\mathrm{P}\left[\mathrm{~N}\left(\mathrm{~d}_{1}\right)\right]-\mathrm{Xe}^{-\mathrm{r}_{\mathrm{Ret}}\left[\mathrm{~N}\left(\mathrm{~d}_{2}\right)\right]}  \tag{9-2}\\
\mathrm{d}_{1} & =\frac{\ln (\mathrm{P} / \mathrm{X})+\left[\mathrm{r}_{\mathrm{RF}}+\left(\sigma^{2} / 2\right)\right] \mathrm{t}}{\sigma \sqrt{\mathrm{t}}}  \tag{9-3}\\
\mathrm{~d}_{2} & =\mathrm{d}_{1}-\sigma \sqrt{\mathrm{t}} . \tag{9-4}
\end{align*}
$$

Here，

$$
\begin{aligned}
\mathrm{V}= & \text { Current value of the call option. } \\
\mathrm{P}= & \text { Current price of the underlying stock. } \\
\mathrm{N}\left(\mathrm{~d}_{\mathrm{i}}\right)= & \text { Probability that a deviation less than } \mathrm{d}_{\mathrm{i}} \text { will occur in a standard } \\
& \text { normal distribution. Thus, } \mathrm{N}\left(\mathrm{~d}_{1}\right) \text { and } \mathrm{N}\left(\mathrm{~d}_{2}\right) \text { represent areas under a } \\
& \text { standard normal distribution function. } \\
\mathrm{X}= & \text { Strike price of the option. } \\
\mathrm{e} \approx & 2.7183 . \\
\mathrm{r}_{\mathrm{RF}}= & \text { Risk-free interest rate. }{ }^{11} \\
\mathrm{t}= & \text { Time until the option expires (the option period). } . \\
\ln (\mathrm{P} / \mathrm{X})= & \text { Natural logarithm of } \mathrm{P} / \mathrm{X} . \\
\sigma^{2}= & \text { Variance of the rate of return on the stock. }
\end{aligned}
$$

Note that the value of the option is a function of the variables we discussed earlier： （1）P，the stock＇s price；（2）t，the option＇s time to expiration；（3）$X$ ，the strike price； （4）$\sigma^{2}$ ，the variance of the underlying stock；and（5） $\mathrm{r}_{\mathrm{RF}}$ ，the risk－free rate．We do not derive the Black－Scholes model－the derivation involves some extremely complicated mathematics that go far beyond the scope of this text．However，it is

[^9]not difficult to use the model. Under the assumptions set forth previously, if the option price is different from the one found by Equation 9-2, this would provide the opportunity for arbitrage profits, which would force the option price back to the value indicated by the model. ${ }^{12}$ As we noted earlier, the Black-Scholes model is widely used by traders, so actual option prices conform reasonably well to values derived from the model.

Loosely speaking, the first term of Equation 9-2, $\mathrm{P}\left[\mathrm{N}\left(\mathrm{d}_{1}\right)\right]$, can be thought of as the expected present value of the terminal stock price, given that $P>X$ and the option will be exercised. The second term, $\mathrm{Xe}^{-\mathrm{r}_{\mathrm{kt}} t}\left[\mathrm{~N}\left(\mathrm{~d}_{2}\right)\right]$, can be thought of as the present value of the exercise price, given that the option will be exercised. However, rather than try to figure out exactly what the equations mean, it is more productive to plug in some numbers to see how changes in the inputs affect the value of an option. The following example is also in the file FM12 Ch 09 Tool Kit.xls.

## OPM Illustration

The current stock price, P , the exercise price, X , and the time to maturity, t , can all be obtained from a newspaper such as The Wall Street Journal. The risk-free rate, $\mathrm{r}_{\mathrm{RF}}$, is the yield on a Treasury bill with a maturity equal to the option expiration date. The annualized variance of stock returns, $\sigma^{2}$, can be estimated by multiplying the variance of the percentage change in daily stock prices for the past year [that is, the variance of $\left(\mathrm{P}_{\mathrm{t}}-\mathrm{P}_{\mathrm{t}-1}\right) / \mathrm{P}_{\mathrm{t}-1}$ ] by 365 days.

Assume that the following information has been obtained:

$$
\begin{aligned}
\mathrm{P} & =\$ 20 . \\
\mathrm{X} & =\$ 20 . \\
\mathrm{t} & =3 \text { months or } 0.25 \text { year. } \\
\mathrm{r}_{\mathrm{RF}} & =6.4 \%=0.064 . \\
\sigma^{2} & =0.16 . \text { Note that if } \sigma^{2}=0.16, \text { then } \sigma=\sqrt{0.16}=0.4
\end{aligned}
$$

Given this information, we can now use the OPM by solving Equations 9-2, 9-3, and 9-4. Since $d_{1}$ and $d_{2}$ are required inputs for Equation $9-2$, we solve Equations 9-3 and 9-4 first:

$$
\begin{aligned}
\mathrm{d}_{1} & =\frac{\ln (\$ 20 / \$ 20)+[0.064+(0.16 / 2)](0.25)}{0.40(0.50)} \\
& =\frac{0+0.036}{0.20}=0.180 \\
\mathrm{~d}_{2} & =\mathrm{d}_{1}-0.4 \sqrt{0.25}=0.180-0.20=-0.020 .
\end{aligned}
$$

Note that $\mathrm{N}\left(\mathrm{d}_{1}\right)=\mathrm{N}(0.180)$ and $\mathrm{N}\left(\mathrm{d}_{2}\right)=\mathrm{N}(-0.020)$ represent areas under a standard normal distribution function. From the table in Appendix D, or from the Excel

e-resource
See FM12 Ch 09 Tool
Kit.xls at the textbook's Web site for all calculations.

[^10]Table 9-2
Effects of OPM Factors on the Value of a Call Option

| Case | INPUT FACTORS |  |  |  |  | $\frac{\text { OUTPUT }}{\mathrm{V}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | P | X | t | $\mathrm{r}_{\text {RF }}$ | $\sigma^{2}$ |  |
| Base case | \$20 | \$20 | 0.25 | 6.4\% | 0.16 | \$ 1.74 |
| Increase P by \$5 | 25 | 20 | 0.25 | 6.4 | 0.16 | 5.57 |
| Increase X by \$5 | 20 | 25 | 0.25 | 6.4 | 0.16 | 0.34 |
| Increase t to 6 months | 20 | 20 | 0.50 | 6.4 | 0.16 | 2.54 |
| Increase $\mathrm{r}_{\text {RF }}$ to 9\% | 20 | 20 | 0.25 | 9.0 | 0.16 | 1.81 |
| Increase $\sigma^{2}$ to 0.25 | 20 | 20 | 0.25 | 6.4 | 0.25 | 2.13 |

function NORMSDIST, we see that the value $\mathrm{d}_{1}=0.180$ implies a probability of $0.0714+0.5000=0.5714$, so $N\left(d_{1}\right)=0.5714$. Since $\mathrm{d}_{2}$ is negative, $\mathrm{N}\left(\mathrm{d}_{2}\right)=0.500-$ $0.0080=0.4920$. We can use those values to solve Equation 9-2:

$$
\begin{aligned}
\mathrm{V} & =\$ 20\left[\mathrm{~N}\left(\mathrm{~d}_{1}\right)\right]-\$ 20 \mathrm{e}^{-(0.064)(0.25)}\left[\mathrm{N}\left(\mathrm{~d}_{2}\right)\right] \\
& =\$ 20[\mathrm{~N}(0.180)]-\$ 20(0.9841)[\mathrm{N}(-0.020)] \\
& =\$ 20(0.5714)-\$ 19.68(0.4920) \\
& =\$ 11.43-\$ 9.69=\$ 1.74
\end{aligned}
$$

Thus the value of the option, under the assumed conditions, is $\$ 1.74$. Suppose the actual option price was $\$ 2.25$. Arbitrageurs could simultaneously sell the option, buy the underlying stock, and earn a riskless profit. Such trading would occur until the price of the option was driven down to $\$ 1.74$. The reverse would occur if the option sold for less than $\$ 1.74$. Thus, investors would be unwilling to pay more than $\$ 1.74$ for the option, and they could not buy it for less, so $\$ 1.74$ is the equilibrium value of the option.

To see how the five OPM factors affect the value of the option, consider Table 9-2. Here the top row shows the base-case input values that were used above to illustrate the OPM and the resulting option value, $\mathrm{V}=\$ 1.74$. In each of the subsequent rows, the boldfaced factor is increased, while the other four are held constant at their base-case levels. The resulting value of the call option is given in the last column. Now let's consider the effects of the changes:

1. Current stock price. If the current stock price, P , increases from $\$ 20$ to $\$ 25$, the option value increases from $\$ 1.74$ to $\$ 5.57$. Thus, the value of the option increases as the stock price increases, but by less than the stock price increase, $\$ 3.83$ versus $\$ 5.00$. Note, though, that the percentage increase in the option value, $(\$ 5.57-\$ 1.74) / \$ 1.74=220 \%$, far exceeds the percentage increase in the stock price, $(\$ 25-\$ 20) / \$ 20=25 \%$.
2. Exercise price. If the exercise price, $X$, increases from $\$ 20$ to $\$ 25$, the value of the option declines. Again, the decrease in the option value is less than the exercise price increase, but the percentage change in the option value, ( $\$ 0.34-$ $\$ 1.74) / \$ 1.74=-78 \%$, exceeds the percentage change in the exercise price, $(\$ 25-\$ 20) / \$ 20=25 \%$.

If an employee stock option grant meets certain conditions, it is called a "tax-qualifying grant," or sometimes an "Incentive Stock Option;" otherwise, it is a "nonqualifying grant." For example, suppose you receive a grant of 1,000 options with an exercise price of $\$ 50$. If the stock price goes to $\$ 110$ and you exercise the options, you must pay $\$ 50(1,000)=$ $\$ 50,000$ for stock that is worth $\$ 110,000$, which is a sweet deal. But what is your tax liability? If you receive a nonqualifying grant, you are liable for ordinary income taxes on $1,000(\$ 110-\$ 50)=$ $\$ 60,000$ when you exercise the option. If it is a taxqualified grant, you owe no regular taxes when exercised. If you then wait at least a year and sell the stock, say, for $\$ 150$, you would have a longterm capital gain of $1,000(\$ 150-\$ 50)=$ $\$ 100,000$, which would be taxed at the lower capital gains rate.

Before you gloat over your newfound wealth, you had better consult your accountant. Your "profit" when you exercise the tax-qualified options isn't taxable under the regular tax code, but it is under the Alternative Minimum Tax (AMT) code. With an AMT tax rate of up to $28 \%$, you might owe as much as $0.28(\$ 110-\$ 50)(1,000)=\$ 16,800$. Here's where people get into trouble. The AMT tax isn't due until the following April, so you might think about waiting until then to sell some stock to pay your AMT
tax (so that the sale will qualify as a long-term capital gain).

But what happens if the stock price falls to $\$ 5$ by next April? You can sell your stock, which raises only $\$ 5(1,000)=\$ 5,000$ in cash. Without getting into the details, you have a long-term capital loss of $1,000(\$ 50-\$ 5)=\$ 45,000$, but IRS regulations limit your net capital loss in a single year to $\$ 3,000$. In other words, the cash from the sale and the tax benefit from the capital loss aren't nearly enough to cover the AMT tax. You may be able to reduce your taxes in future years because of the AMT tax you pay this year and the carry forward of the remaining long-term capital loss, but that doesn't help right now. You lost $\$ 45,000$ of your original $\$ 50,000$ investment, you now have very little cash, and, adding insult to injury, the IRS will insist that you also pay the \$16,800 AMT tax.

This is exactly what happened to many people who made paper fortunes in the dot-com boom only to see them evaporate in the ensuing bust. They were left with worthless stock but multi-million-dollar AMT tax obligations. In fact, many still have IRS liens garnishing their wages until they eventually pay their AMT tax. So if you receive stock options, we congratulate you. But unless you want to be the next poster child for poor financial planning, we advise you to settle your AMT tax when you incur it.
3. Option period. As the time to expiration increases from $t=3$ months (or 0.25 year) to $t=6$ months (or 0.50 year), the value of the option increases from $\$ 1.74$ to $\$ 2.54$. This occurs because the value of the option depends on the chances for an increase in the price of the underlying stock, and the longer the option has to go, the higher the stock price may climb. Thus, a 6-month option is worth more than a 3 -month option.
4. Risk-free rate. As the risk-free rate increases from $6.4 \%$ to $9 \%$, the value of the option increases slightly, from $\$ 1.74$ to $\$ 1.81$. Equations $9-1,9-2$, and $9-3$ suggest that the principal effect of an increase in $\mathrm{r}_{\mathrm{RF}}$ is to reduce the present value of the exercise price, $\mathrm{Xe}^{-r_{\mathrm{w}} \mathrm{t}}$, ,hence to increase the current value of the option. ${ }^{13}$ The risk-free rate also plays a role in determining the values of the normal distribution functions $N\left(d_{1}\right)$ and $N\left(d_{2}\right)$, but this effect is of secondary importance. Indeed, option prices in general are not very sensitive to interest rate changes, at least not to changes within the ranges normally encountered.

[^11]5. Variance. As the variance increases from the base case 0.16 to 0.25 , the value of the option increases from $\$ 1.74$ to $\$ 2.13$. Therefore, the riskier the underlying security, the more valuable the option. This result is logical. First, if you bought an option to buy a stock that sells at its exercise price, and if $\sigma^{2}=0$, then there would be a zero probability of the stock going up, hence a zero probability of making money on the option. On the other hand, if you bought an option on a high-variance stock, there would be a higher probability that the stock would go way up, hence that you would make a large profit on the option. Of course, a high-variance stock could go way down, but as an option holder, your losses would be limited to the price paid for the option-only the right-hand side of the stock's probability distribution counts. Put another way, an increase in the price of the stock helps option holders more than a decrease hurts them, so the greater the variance, the greater the value of the option. This makes options on risky stocks more valuable than those on safer, low-variance stocks.

Myron Scholes and Robert Merton were awarded the 1997 Nobel Prize in Economics, and Fischer Black would have been a co-recipient had he still been living. Their work provided analytical tools and methodologies that are widely used to solve many types of financial problems, not just option pricing. Indeed, the entire field of modern risk management is based primarily on their contributions. Although the Black-Scholes model was derived for a European option that can be exercised only on its maturity date, it also applies to American options that don't pay any dividends prior to expiration. The textbooks by Don Chance and John Hull that we listed in Footnote 2 show adjusted models for dividend-paying stocks.

## SELF-TEST

What is the purpose of the Black-Scholes Option Pricing Model?
Explain what a "riskless hedge" is and how the riskless hedge concept is used in the Black-Scholes OPM.
Describe the effect of a change in each of the following factors on the value of a call option: (1) stock price; (2) exercise price; (3) option life; (4) risk-free rate; and (5) stock return variance, that is, risk of stock.
What is the value of a call option with these data: $P=\$ 35, X=\$ 25, r_{R F}=6 \%, t=0.5$ ( 6 months), and $\sigma^{2}=0.36$ ? (\$12.05)

### 9.4 The Valuation of Put Options

A put option gives its owner the right to sell a share of stock. If the stock pays no dividends and the option can only be exercised upon its expiration date, what is its value? Rather than reinventing the wheel, consider the payoffs for two portfolios at expiration date T , as shown in Table 9-3. The first portfolio consists of a put option and a share of stock; the second has a call option (with the same strike price and expiration date as the put option) and some cash. The amount of cash is equal to the present value of the exercise cost, discounted at the continuously compounded risk-free rate, which is $\mathrm{Xe}^{-\mathrm{r}_{\mathrm{Rt}} \mathrm{t}}$. At expiration, the value of this cash will equal the exercise cost, X .

If the stock price at expiration date $\mathrm{T}, \mathrm{P}_{\mathrm{T}}$, is less than the strike price, X, when the option expires, then the value of the put option at expiration is $X-P_{T}$. Therefore, the value of Portfolio 1, which contains the put and the stock, is equal to $X-P_{T}$ plus $P_{T}$, or just $X$. For Portfolio 2, the value of the call is zero at expiration

(because the call option is out-of-the-money), and the value of the cash is $X$, for a total value of $X$. Notice that both portfolios have the same payoffs if the stock price is less than the strike price.

What if the stock price is greater than the strike price at expiration? In this case, the put is worth nothing, so the payoff of Portfolio 1 is equal to the stock price at expiration, $\mathrm{P}_{\mathrm{T}}$. The call option is worth $\mathrm{P}_{\mathrm{T}}-\mathrm{X}$, and the cash is worth X , so the payoff of Portfolio 2 is $\mathrm{P}_{\mathrm{T}}$. Therefore, the payoffs of the two portfolios are equal, whether the stock price is below or above the strike price.

If the two portfolios have identical payoffs, they must have identical values. This is known as the put-call parity relationship:

$$
\text { Put option }+ \text { Stock }=\text { Call option }+ \text { PV of exercise price. }
$$

If V is the Black-Scholes value of the call option, then the value of a put is ${ }^{14}$

$$
\begin{equation*}
\text { Put option }=\mathrm{V}-\mathrm{P}+\mathrm{Xe}^{-\mathrm{r}_{\mathrm{Rr}} \mathrm{t}} \text {. } \tag{9-5}
\end{equation*}
$$

For example, consider a put option written on the stock discussed in the previous section. If the put option has the same exercise price and expiration date as the call, its price is

$$
\begin{aligned}
& \text { Put option }=\$ 1.74-\$ 20+\$ 20 \mathrm{e}^{-0.064(0.25)} \\
& \quad=\$ 1.74-\$ 20+\$ 19.68=1.42
\end{aligned}
$$

## SELF-TEST

In words, what is put-call parity?
A put option written on the stock of Taylor Enterprises (TE) has an exercise price of $\$ 25$ and 6 months remaining until expiration. The risk-free rate is $6 \%$. A call option written on TE has the same exercise price and expiration date as the put option. TE's stock price is $\$ 35$. If the call option has a price of $\$ 12.05$, what is the price (i.e., value) of the put option? (\$1.26)

[^12]
### 9.5 Applications of Option Pricing in Corporate Finance

Option pricing is used in four major areas in corporate finance: (1) real options analysis for project evaluation and strategic decisions, (2) risk management, (3) capital structure decisions, and (4) compensation plans.

## Real Options

Suppose a company has a 1-year proprietary license to develop a software application for use in a new generation of wireless cellular telephones. Hiring programmers and marketing consultants to complete the project will cost $\$ 30$ million. The good news is that if consumers love the new cell phones, there will be a tremendous demand for the new software. The bad news is that if sales of the new cell phones are low, the software project will be a disaster. Should the company spend the $\$ 30$ million and develop the software?

Because the company has a license, it has the option of waiting for a year, at which time it might have a much better insight into market demand for the new cell phones. If demand is high in a year, then the company can spend the $\$ 30$ million and develop the software. If demand is low, it can avoid losing the $\$ 30$ million development cost by simply letting the license expire. Notice that the license is analogous to a call option: It gives the company the right to buy something (in this case, software for the new cell phones) at a fixed price ( $\$ 30$ million) at any time during the next year. The license gives the company a real option, because the underlying asset (the software) is a real asset and not a financial asset.

There are many other types of real options, including the option to increase capacity at a plant, to expand into new geographical regions, to introduce new products, to switch inputs (such as gas versus oil), to switch outputs (such as producing sedans versus SUVs), and to abandon a project. Many companies now evaluate real options with techniques similar to those described earlier in the chapter for pricing financial options. Real options are described in greater depth in Chapter 13.

## Risk Management

Suppose a company plans to issue $\$ 400$ million of bonds in 6 months to pay for a new plant now under construction. The plant will be profitable if interest rates remain at current levels, but if rates rise, it will be unprofitable. To hedge against rising rates, the company could purchase a put option on Treasury bonds. If interest rates go up, the company would "lose" because its bonds would carry a high interest rate, but it would have an offsetting gain on its put options. Conversely, if rates fall, the company would "win" when it issues its own low-rate bonds, but it would lose on the put options. By purchasing puts, the company has hedged the risk it would otherwise face due to possible interest rate changes.

Another example of risk management is a firm that bids on a foreign contract. For example, suppose a winning bid means the firm will receive a payment of 12 million euros in 9 months. At a current exchange rate of $\$ 1.04$ per euro, the project would be profitable. But if the exchange rate falls to $\$ 0.80$ per euro, the project would be a loser. To avoid exchange rate risk, the firm could take a short position in a forward contract, which would allow it to convert 12 million euros into
dollars at a fixed rate of $\$ 1.00$ per euro in 9 months, which would still ensure a profitable project. This eliminates exchange rate risk if the firm wins the contract, but what if the firm loses the contract? It would still be obligated to sell 12 million euros at a price of $\$ 1.00$ per euro, which could be a disaster. For example, if the exchange rate rises to $\$ 1.25$ per euro, the firm would have to spend $\$ 15$ million to purchase 12 million euros at a price of $\$ 1.25 / €$ and then sell the euros for $\$ 12$ million $=(\$ 1.00 / €)(€ 12$ million $)$, a loss of $\$ 3$ million.

To eliminate this risk, the firm could also purchase a currency call option that allows it to buy 12 million euros at a fixed price of $\$ 1.00$ per euro. If the company wins the bid, it will let the option expire, but it will use the forward contract to convert the euros at the forward contract's rate of $\$ 1.00$ per euro. If the firm loses the bid, then it will exercise the call option and purchase 12 million euros for $\$ 1.00$ per euro. It will then use those proceeds to close out the forward contract. Thus, the company is able to lock in the future exchange rate if it wins the bid and avoid any net payments at all if it loses the bid. The total cost in either scenario is equal to the initial cost of the option. In other words, the cost of the option is like insurance that guarantees the exchange rate if the company wins the bid and guarantees no net obligations if it loses the bid.

Many other applications of risk management involve futures contracts and other complex derivatives rather than calls and puts. However, the principles used in pricing derivatives are similar to those used earlier in this chapter for pricing options. Thus, financial options and their valuation techniques play key roles in risk management. Derivatives and their use in risk management are discussed in greater depth in Chapter 23.

## Capital Structure Decisions

Decisions regarding the mix of debt and equity used to finance operations are quite important. One interesting aspect of the capital structure decision is based on option pricing. For example, consider a firm with debt requiring a final principal payment of $\$ 60$ million in 1 year. If the company's value 1 year from now is $\$ 61$ million, then it can pay off the debt and have $\$ 1$ million left for stockholders. If the firm's value is less than $\$ 60$ million, then it might well file for bankruptcy and turn over its assets to the creditors, resulting in stockholders' equity of zero. In other words, the value of the stockholders' equity is analogous to a call option: The equity holders have the right to buy the assets for $\$ 60$ million (which is the face value of the debt) in 1 year (when the debt matures).

Suppose the firm's owner-managers are considering two projects. One has very little risk, and it will result in an asset value of either $\$ 59$ million or $\$ 61$ million. The other has high risk, and it will result in an asset value of either $\$ 20$ million or $\$ 100$ million. Notice that the equity will be worth zero if the assets are worth less than $\$ 60$ million, so the stockholders will be hurt no worse if the assets end up at $\$ 20$ million than if they end up at $\$ 59$ million. On the other hand, the stockholders would benefit much more if the assets were worth $\$ 100$ million rather than $\$ 61$ million. Thus, the owner-managers have an incentive to choose risky projects, which is consistent with an option's value rising with the risk of the underlying asset. Potential lenders recognize this situation, so they build covenants into loan agreements that restrict managers from making excessively risky investments.

Not only does option pricing theory help explain why managers might want to choose risky projects (for example, think about Enron) and why debtholders might want very restrictive covenants, but options also play a direct role in capital structure choices. For example, a firm might choose to issue convertible debt,
which gives bondholders the option to convert their debt into stock if the value of the company turns out to be higher than expected. In exchange for this option, bondholders charge a lower interest rate than for nonconvertible debt. Because owner-managers must share the wealth with convertible-bond holders, they have a smaller incentive to gamble with high-risk projects. We discuss options and capital structure in Chapter 17 and convertible securities in Chapter 21.

## Compensation Plans

Many companies use stock options as a part of their compensation plans. It is important for boards of directors to understand the value of these options before they grant them to employees. We discuss compensation issues associated with stock options in more detail in Chapter 15.

Describe four ways that option pricing is used in corporate finance.

## Summary

In this chapter we discussed option pricing topics, including the following:

- Financial options are instruments that (1) are created by exchanges rather than firms, (2) are bought and sold primarily by investors, and (3) are of importance to both investors and financial managers.
- The two primary types of financial options are (1) call options, which give the holder the right to purchase a specified asset at a given price (the exercise, or strike, price) for a given period of time, and (2) put options, which give the holder the right to sell an asset at a given price for a given period of time.
- A call option's exercise value is defined as the maximum of zero or the current price of the stock less the strike price.
- The Black-Scholes Option Pricing Model (OPM) can be used to estimate the value of a call option.
- The five inputs to the Black-Scholes model are (1) P, the current stock price; (2) $X$, the strike price; (3) $r_{\text {RF }}$, the risk-free interest rate; (4) $t$ t the remaining time until expiration; and (5) $\sigma^{2}$, the variance of the stock's rate of return.
- A call option's value increases if: $P$ increases, $X$ decreases, $r_{R F}$ increases, $t$ increases, or $\sigma^{2}$ increases.
- The put-call parity relationship states that: Put option + Stock $=$ Call option + PV of exercise price.


## Questions

(9-1) Define each of the following terms:
a. Option; call option; put option
b. Exercise value; strike price
c. Black-Scholes Option Pricing Model
(9-2) Why do options sell at prices higher than their exercise values?
(9-3) Describe the effect on a call option's price caused by an increase in each of the following factors: (1) stock price, (2) strike price, (3) time to expiration, (4) risk-free rate, and (5) variance of stock return.

## Self-Test Problems

## Solutions Appear in Appendix A

(ST- 1) The current price of a stock is $\$ 40$. In 1 year, the price will be either $\$ 60$ or $\$ 30$. The annual risk-free rate is $5 \%$. Find the price of a call option on the stock that has a strike price of $\$ 42$ and that expires in 1 year. (Hint: Use daily compounding.)
(ST-2) Use the Black-Scholes model to find the price for a call option with the following Black-Scholes Model inputs: (1) current stock price is $\$ 22$, (2) strike price is $\$ 20$, (3) time to expiration is 6 months, (4) annualized risk-free rate is $5 \%$, and (5) variance of stock return is 0.49 .

## Problems

## Answers Appear in Appendix B

## Easy

Problems 1-2
(9-1) A call option on the stock of Bedrock Boulders has a market price of $\$ 7$. The stock Options sells for $\$ 30$ a share, and the option has a strike price of $\$ 25$ a share. What is the exercise value of the call option? What is the option's time value?
(9-2) The exercise price on one of Flanagan Company's options is $\$ 15$, its exercise value is $\$ 22$, and its time value is $\$ 5$. What are the option's market value and the price of the stock?

## Intermediate

Problems 3-4
(9-3) Assume you have been given the following information on Purcell Industries:
Black-Scholes Model

$$
\begin{array}{ll}
\text { Current stock price }=\$ 15 & \text { Strike price of option }=\$ 15 \\
\text { Time to maturity of option }=6 \text { months } & \text { Risk-free rate }=6 \% \\
\text { Variance of stock return }=0.12 & \mathrm{~d}_{1}=0.24495 \\
\mathrm{~d}_{2}=0.00000 & \mathrm{~N}\left(\mathrm{~d}_{1}\right)=0.59675
\end{array}
$$

$$
\mathrm{N}\left(\mathrm{~d}_{2}\right)=0.50000
$$

Using the Black-Scholes Option Pricing Model, what would be the value of the option?
(9-4) The current price of a stock is $\$ 33$, and the annual risk-free rate is $6 \%$. A call option with a strike price of $\$ 32$ and 1 year until expiration has a current value of $\$ 6.56$. What is the value of a put option written on the stock with the same strike price and expiration date as the call option?

## Challenging <br> Problems 5-7

(9-5) Use the Black-Scholes model to find the price for a call option with the following Black-Scholes Model inputs: (1) current stock price is $\$ 30$, (2) strike price is $\$ 35$, (3) time to expiration is 4 months, (4) annualized risk-free rate is $5 \%$, and (5) variance of stock return is 0.25 .
(9-6) The current price of a stock is $\$ 20$. In 1 year, the price will be either $\$ 26$ or Binomial Model $\$ 16$. The annual risk-free rate is $5 \%$. Find the price of a call option on the stock that has a strike price of $\$ 21$ and that expires in 1 year. (Hint: Use daily compounding.)
(9-7) The current price of a stock is $\$ 15$. In 6 months, the price will be either $\$ 18$ or Binomial Model $\$ 13$. The annual risk-free rate is $6 \%$. Find the price of a call option on the stock that has a strike price of $\$ 14$ and that expires in 6 months. (Hint: Use daily compounding.)

## Spreadsheet Problem

(9-8) Start with the partial model in the file FM12 Ch 09 P08 Build a Model.xls from the textbook's Web site. Rework Problem 9-3. Then work the next two parts of this Scholes Model problem given below.
a. Construct data tables for the exercise value and Black-Scholes option value for this option, and graph this relationship. Include possible stock price values ranging up to $\$ 30.00$.
b. Suppose this call option is purchased today. Draw the profit diagram of this option position at expiration.

## Cyberproblem

Please go to the textbook's Web site to access any Cyberproblems.

## Mini Case

Assume that you have just been hired as a financial analyst by Triple Trice Inc., a mid-sized California company that specializes in creating exotic clothing. Because no one at Triple Trice is familiar with the basics of financial options, you have been asked to prepare a brief report that the firm's executives can use to gain at least a cursory understanding of the topic.

To begin, you gathered some outside materials on the subject and used these materials to draft a list of pertinent questions that need to be answered. In fact, one possible approach to the paper is to use a question-and-answer format. Now that the questions have been drafted, you have to develop the answers.
a. What is a financial option? What is the single most important characteristic of an option?
b. Options have a unique set of terminology. Define the following terms:
(1) Call option
(2) Put option
(3) Exercise price
(4) Striking, or strike, price
(5) Option price
(6) Expiration date
(7) Exercise value
(8) Covered option
(9) Naked option
(10) In-the-money call
(11) Out-of-the-money call
(12) LEAPS
c. Consider Triple Trice's call option with a $\$ 25$ strike price. The following table contains historical values for this option at different stock prices:

| Stock Price | Call Option Price |
| :---: | :---: |
| $\$ 25$ | $\$ 3.00$ |
| 30 | 7.50 |
| 35 | 12.00 |
| 40 | 16.50 |
| 45 | 21.00 |
| 50 | 25.50 |

(1) Create a table that shows (a) stock price, (b) strike price, (c) exercise value, (d) option price, and (e) the time value, which is the option's price less its exercise value.
(2) What happens to the time value as the stock price rises? Why?
d. In 1973, Fischer Black and Myron Scholes developed the Black-Scholes Option Pricing Model (OPM).
(1) What assumptions underlie the OPM?
(2) Write out the three equations that constitute the model.
(3) What is the value of the following call option according to the OPM?

Stock price $=\$ 27.00$
Strike price $=\$ 25.00$
Time to expiration $=6$ months
Risk-free rate $=6.0 \%$
Stock return variance $=0.11$
e. What impact does each of the following call option parameters have on the value of a call option?
(1) Current stock price
(2) Strike price
(3) Option's term to maturity
(4) Risk-free rate
(5) Variability of the stock price
f. What is put-call parity?


[^0]:    Interestingly, Microsoft and some other companies stopped granting options after a change in accounting regulations that requires that the estimated value of options be "expensed," or taken as a cost at the time they are granted. We discuss expensing options later in this chapter.

[^1]:    ${ }^{2}$ For an in-depth treatment of options, see Don M. Chance, An Introduction to Derivatives and Risk Management (Mason, OH: Thomson/South-Western, 2004) or John C. Hull, Options, Futures, and Other Derivatives (Upper Saddle River, NJ: Prentice-Hall, 2006).

[^2]:    ${ }^{3}$ The expiration date is the Friday before the third Saturday of the exercise month. Also, note that option contracts are generally written in 100-share multiples.

[^3]:    ${ }^{4}$ Insiders who trade illegally generally buy options rather than stock because the leverage inherent in options increases the profit potential. Note, though, that it is illegal to use insider information for personal gain, and an insider using such information would be taking advantage of the option seller. Insider trading, in addition to being unfair and essentially equivalent to stealing, hurts the economy: Investors lose confidence in the capital markets and raise their required returns because of an increased element of risk, and this raises the cost of capital and thus reduces the level of real investment.
    ${ }^{5} \operatorname{MAX}$ means choose the maximum. For example, $\operatorname{MAX}[15,0]=15$, and $\operatorname{MAX}[-10,0]=0$.

[^4]:    "Among traders an option's market price is also called its "premium." This is particularly confusing since for all other securities the term premium means the excess of the market price over some base price. To avoid confusion, we will not use the term premium to refer to the option price. Also, the difference between an option's market price and its exercise value is called its "time value" because this represents the extra amount over the option's immediate exercise value a purchaser will pay for the chance the stock price will appreciate over time.

[^5]:    ${ }^{7}$ As we'll soon show, we don't have to specify the probability of the ending prices to calculate the option price

[^6]:    ${ }^{8}$ To be technically correct, we should discount the ending value using a continuously compounded interest rate, as discussed in Web Extension 2B.

[^7]:    ${ }^{9}$ Suppose an investor (or speculator) does not own any IBM stock. If the investor anticipates a rise in the stock price and consequently buys IBM stock, he or she is said to have gone long in IBM. On the other hand, if the investor thinks IBM's stock is likely to fall, he or she could go short, or sell IBM short. Because the short seller has no IBM stock, he or she would have to borrow the shares from a broker and sell the borrowed shares. If the stock price falls, the short seller could, later on, buy shares on the open market and pay back the ones borrowed from the broker. The short seller's profit, before commissions and taxes, would be the difference between the price received from the short sale and the price paid later to purchase the replacement stock.

[^8]:    ${ }^{10}$ See the books by Chance or Hull, cited in Footnote 2.

[^9]:    ${ }^{11}$ The risk－free rate should be expressed as a continuously compounded rate．If $r$ is a continuously compounded rate，then the effective annual yield is $\mathrm{e}^{r}-1.0$ ．An $8 \%$ continuously compounded rate of return yields $\mathrm{e}^{0.08}-1=$ $8.33 \%$ ．In all of the Black－Scholes option pricing model examples，we will assume that the rate is expressed as a con－ tinuously compounded rate．

[^10]:    ${ }^{12}$ Programmed trading, in which stocks are bought and options are sold, or vice versa, is an example of arbitrage between stocks and options.

[^11]:    ${ }^{13}$ At this point, you may be wondering why the first term in Equation 9-2, $\mathrm{P}\left[\mathrm{N}\left(\mathrm{d}_{1}\right)\right]$, is not discounted. In fact, it has been, because the current stock price, $P$, already represents the present value of the expected stock price at expiration. In other words, P is a discounted value, and the discount rate used in the market to determine today's stock price includes the riskfree rate. Thus, Equation 9-2 can be thought of as the present value of the end-of-period-option spread between the stock price and the strike price, adjusted for the probability that the stock price will be higher than the strike price.

[^12]:    ${ }^{14}$ This model cannot be applied to an American put option or to a European option on a stock that pays a dividend prior to expiration. For an explanation of valuation approaches in these situations, see the books by Chance or Hull cited in Footnote 2.

